Pergamon

Int. J. Multiphase Flow Vol. 20, No. 5, pp. 969–977, 1994
Elsevier Science Ltd. Printed in Great Britain
0301-9322(94)E0002-Z

THE INFLUENCE OF A MEAN FLUID VELOCITY GRADIENT ON THE PARTICLE–FLUID VELOCITY COVARIANCE

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(Received 3 August 1993; in revised form 27 December 1993)

Abstract—The effect of a mean fluid velocity gradient on the particle–fluid velocity covariance and fluctuating relative velocity for a small solid particle suspended in a turbulent gas is examined using Fourier transform techniques. The presence of such a gradient is shown to elevate the covariance above the level predicted without it. The variance of the fluctuating relative velocity is not directly affected by the presence of a mean velocity gradient. The possible impact on turbulence modulation by particles is discussed.

Key Words: particulate motion, turbulence, particle velocity variance, two-phase flow

1. INTRODUCTION

The purpose of this analysis is to demonstrate that the magnitude of the velocity covariance can be elevated by the presence of mean velocity gradients in the flow. The variance in the fluctuating relative velocity is not directly affected by the presence of a mean velocity gradient. A major goal is to illustrate that analyses that neglect the mean velocity gradients may fail to capture an essential feature of particle–fluid covariance: the covariance may exceed the variance of the fluctuating fluid velocity. This suggests that even when the flow around the particle corresponds to Stokes flow, particle–fluid interactions may represent a source term that generates turbulence in the fluid phase.

Quantitiative solutions for the magnitude of the particle–fluid velocity covariance will differ depending on the exact flow field, its turbulence characteristics and even the direction of the gravitational vector relative to the flow field. For this reason, an idealized flow field was selected to illustrate the effect of mean velocity gradients on the velocity covariance; this makes the analysis qualitative in the sense that it does not provide a quantitative prediction of the magnitude of the covariance for any particular flow field. Caution should be used when attempting to apply the results to predicting the magnitude of the covariance in real flows.

Although the mechanism is described in a qualitative way, the description is important because complete solutions of the particle velocity statistics in a turbulent field require relatively complex analyses. They are warranted only when all important mechanisms are recognized and incorporated into the solution. Moreover, predicting particle statistics is essential to predicting transport properties such as particle dispersion, and trubulence modulation by particles. For example, a large fluctuating relative velocity causes a loss of correlation between a particle and the fluid packet in which it resides, and is known to have a large impact on dispersion rates in many applications (Lumley 1957). Consequently, features that affect this statistic may influence dispersion rates significantly. In addition, it has been proposed that the relative magnitude of the fluctuating kinetic energies of the fluid and particle phases and the particle–fluid velocity covariance can lead to dissipation of turbulent kinetic energy in the fluid phase of particulate flows (Elghobashi & Abou-Arab 1983; Besnard & Harlow 1988; Rogers & Eaton 1991).

It is recognized that other phenomena that are not described in this paper could also affect the magnitude of the particle velocity covariance and other velocity moments. However, detailed analyses to predict these moments will be useful only after all physical phenomena that contribute significantly to their magnitude have been identified. The goal of the analysis illustrates that models

†Operated for the U.S. Department of Energy by Battelle Memorial Institute under Contract DE-AC06-76RL0-1830.

based on homogeneous-isotropic turbulence (HIT) fail to account for an effect that can be important in some flows. The goal is not to propose a universal model for the particle statistics.

The organization of this paper is as follows: a qualitative analysis of the effect of the mean velocity gradient on the velocity covariance and fluctuating relative velocity is provided in section 2; this analysis is similar in form to that contained in Liljegren (1993), which dealt with the particle velocity variance only. A discussion of the effect of particles on the generation or dissipation of turbulence follows in section 3.

2. QUALITATIVE ANALYSIS OF THE EFFECT OF MEAN VELOCITY GRADIENTS ON COVARIANCE

The motion of a small solid spherical particle suspended in an incompressible, unbounded, homogeneous turbulent flow will now be considered. The mean fluid velocity will be assumed to be plane parallel in the x (or streamwise) coordinate direction and to vary spatially with constant velocity gradient, G(i.e. $\bar{U}_x = Gy$, $\bar{U}_y = 0$, and $\bar{U}_z = 0$). The fluid turbulence will be assumed to be homogeneous and stationary; this is artificial, but no more artificial than the assumption of stationary homogeneous, isotropic turbulence. A simplified form of the turbulence spectra will be used, and the effect of a mean velocity drift between the particle and fluid will be neglected when evaluating the turbulence spectrum. The justification for both these simplifying assumptions is provided in Liljegren (1990, 1993). It is sufficient to state here that the assumptions are permissible provided that the result obtained here is considered to be qualitative rather than quantitative.

The equations governing the transverse particle velocity V_y and the streamwise particle velocity V_x as seen in a Lagrangian framework following the particle are

$$\dot{V}_x + \beta V_x = \beta(GY_p + u_x)_{|x_p} + \beta V_{sy} \quad [1]$$

and

$$\dot{V}_y + \beta V_y = \beta u_{y|x_p} + \beta V_{sx} \quad [2]$$

where V_{sx} and V_{sy} are the x and y components of the terminal settling velocity, β is the inverse particle relaxation time, Y_p is the current particle y coordinate and u_x and u_y are the local velocity fluctuations for the fluid phase.

When Stokes equations describe settling, these are $V_{sx} = \Delta\rho g_x/\rho\beta$ and $V_{sy} = \Delta\rho g_x/\rho\beta$; for a particle with radius a, $\beta = 9\mu/2\rho_p a^2$.

The average location of the particle may be determined by taking the ensemble average of [1] and [2]. After the initial transient, the expected value of the particle velocities will become $\overline{V_y} = V_{sy}t$ and $\overline{V_x} = \beta V_{sy} - GV_{sx}$; the expected location of the particle will be $\overline{X_p} = 0$ and $\overline{Y_p} = V_{sy}t$.

The equations governing the transverse particle velocity fluctuations v_y and streamwise particle velocity fluctuation v_x may be obtained by subtracting the ensemble average contributions to [1] and [2]. This results in

$$\dot{v}_x + \beta v_x = \beta(Gy_p + u_x)_{|x_p} \quad [3]$$

and

$$\dot{v}_y + \beta v_y = \beta u_{y|x_p} \quad [4]$$

The transport equations [3] and [4] shown above are somewhat inconvenient for the purpose of this analysis. This is primarily because [3] and [4] describe the particle velocity in a Lagrangian framework, and [3] is not statistically stationary. This difficulty can be solved by defining a new velocity:

$$w_x = v_x - \bar{U}_x(y_p) = v_x - Gy_p \quad [5]$$

The quantity w_x describes the difference between the velocity of a particle and the mean velocity of the fluid at the particle's current position. The quantity, $\Delta w_x = w_x - u_x$ describes the instantaneous relative velocity between a particle and the packet of fluid that contains that particle. Consequently, features such as the particle Reynolds number, and instantaneous interfacial force between phases can be described more easily using w_x than v_x. In addition, w_x is the quantity measured in experiments including those by Soo *et al.* (1960), Carlson & Peskin (1975), Steimke & Dukler (1983), Tsuji & Morikawa (1982), Rogers & Eaton (1990) and Liljegren (1990).

Recognizing that $\mathrm{d}y_\mathrm{p}/\mathrm{d}t = v_y$ and using [5] and [3] results in

$$\dot{w}_x + \beta w_x = -Gv_y + \beta u_x \qquad [6]$$

Equations [4] and [6] have been analyzed (Liljegren 1990, 1993) to determine the effect of the mean velocity gradient on the particle velocity variances $\overline{v_y v_y}$ and $\overline{w_x w_x}$ of particles. Knowing that, it is possible to determine the magnitude of the velocity covariance by first defining a "fluctuating relative" velocity as

$$\Delta w_x = w_x - u_x \quad \text{and} \quad \Delta v_y = v_y - u_y \qquad [7]$$

and then determining the magnitude of the covariance algebraically using

$$2\overline{u_y v_y} = \overline{u_y u_y} + \overline{v_y v_y} - \overline{\Delta v_y \, \Delta v_y} \qquad [8]$$

and

$$2\overline{u_x w_x} = \overline{u_x u_x} + \overline{w_x w_x} - \overline{\Delta w_x \, \Delta w_x} \qquad [9]$$

The equations governing the two components of the fluctuating relative velocity are

$$\Delta \dot{v}_y + \beta \, \Delta v_y = \dot{u}_{y|x_\mathrm{p}} \qquad [10]$$

and

$$\Delta \dot{w}_x + \beta \, \Delta w_x = \{-Gv_y + \dot{u}_x\}_{|x_\mathrm{p}} \qquad [11]$$

Let the Fourier transform of a velocity $\hat{V}_i$ and its inverse be defined:

$$\hat{V}_i(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-i\omega t} V_i(t)\,\mathrm{d}t \quad \text{and} \quad V_i(t) = \int_{-\infty}^{\infty} \mathrm{e}^{i\omega t}\hat{V}_i(\omega)\,\mathrm{d}\omega \qquad [12]$$

The spectra of the two components of the relative velocity defined as the Fourier transform of the respective velocity autocorrelations, are then

$$S_{\Delta v_y \Delta v_y}(\omega) = \frac{\omega^2}{\beta^2 + \omega^2} S_{u_y u_y}(\omega) \qquad [13]$$

and

$$S_{\Delta w_x \Delta w_x} = \frac{\omega^2}{\beta^2 + \omega^2} S_{u_x u_x} + \frac{i\omega G}{\beta^2 + \omega^2}(S_{u_x v_y} - S^*_{u_x v_y}) + \frac{G^2}{\beta^2 + \omega^2} S_{v_y v_y} \qquad [14]$$

where S_{yy} denotes a component of a velocity spectrum tensor; the particular component is described by the form of the subscripts.

The velocity variance can be determined by integrating the spectra, resulting in

$$\overline{\Delta v_y \, \Delta v_y} = I_{4y} \qquad [15]$$

and

$$\overline{\Delta w_x \, \Delta w_x} = I_{4x} + I_{5x} + I_{3x} \qquad [16]$$

where

$$I_{4y} = 2\int_0^\infty \frac{(\omega/\beta)^2}{1 + (\omega/\beta)^2} S_{u_y u_y}(\omega)\,\mathrm{d}\omega \qquad [17]$$

$$I_{4x} = 2\int_0^\infty \frac{(\omega/\beta)^2}{1 + (\omega/\beta)^2} S_{u_x u_x}(\omega)\,\mathrm{d}\omega \qquad [18]$$

$$I_{5x} = -2\frac{G}{\beta}\int_0^\infty \left\{\frac{(\omega/\beta)}{1 + (\omega/\beta)^2}\right\}^2 2\,\mathrm{Re}\,(S_{u_x u_y}(\omega))\,\mathrm{d}\omega \qquad [19]$$

and

$$I_{3x} = 2\frac{G^2}{\beta^2}\int_0^\infty \left\{\frac{1}{1+(\omega/\beta)^2}\right\}^2 S_{u_y u_y}(\omega)\,d\omega \qquad [20]$$

Rigorous evaluation of [17] and [20] requires specification of the fluid velocity spectrum as sampled along the particle path. Because the form of the spectra depends on specific flow features, it is difficult to obtain a general result that describes the particle motion for all flows. Moreover, because the particle path may only be described in a stochastic sense and is itself a function of the random particle position, the problem is inherently non-linear (Corrsin & Lumley 1956; Lumley 1957). Methods to account for the non-linearity have been described by Reeks (1977) and Nir & Pismen (1979). Although the non-linearity is of great importance when attempting to obtain quantitative results, it will be ignored here because it does not affect the major goal of the analysis, which is to show that the accounting for the mean velocity gradients can be essential to predicting particle statistics. In some cases, accounting for the velocity gradients will be at least as important as accounting for the non-linearity inherent to the problem.

When the particle Stokes number is small, the spectrum seen along the path of an individual particle is approximately equal to the Lagrangian fluid velocity spectrum. The exact form of this spectrum in a region with mean velocity gradients is not completely understood. So, for the purposes of this analysis, a spectral form recommended for homogeneous isotropic turbulence by Tennekes & Lumley (1972) will be used.

The diagonal components are assumed to be of the form

$$S_{u_x u_x} = S_{u_y u_y} = \mathrm{uL}\, X_{ii}/3 = \mathrm{u}^2\pi/4\omega_e X_{ii} \qquad [21]$$

where

$$X_{ii}(\omega) = \pi^{-1} \quad \text{for } |\omega \mathrm{L/u}| < \frac{3\pi}{4}$$

and

$$X_{ii}(\omega) = \pi^{-1}(\omega_e/\omega)^2 \quad \text{for } \frac{3\pi}{4} < |\omega \mathrm{L/u}| < \infty$$

and

$$\omega_e = \frac{3\pi}{4}\mathrm{u/L}$$

L is the Lagrangian integral scale of the turbulence, and u is the characteristic fluid velocity fluctuation. When the turbulence is isotropic, the following relation holds:

$$\mathrm{u}^2 = \overline{u_x u_x} = \overline{u_y u_y} \qquad [22]$$

Although the spectrum is known to be anisotropic in flows with mean velocity gradients, the spectrum described above is isotropic. The consequence of this assumption will be discussed at the end of this section. A more detailed description of the spectrum and the uncertainties associated with its use is provided in the author's previous analysis (Liljegren 1990, 1993).

In Liljegren (1990 and 1993) the leading-order contributions from the mean velocity gradient to the velocity variances were shown to be $O(\alpha)$, we may determine the effect of the mean velocity gradients on the covariance to consistent order by evaluating the terms that contribute to the variance to $O(\alpha)$; higher order terms $O(\alpha^2)$ may be ignored. The terms I_{3y}, I_{3x} and I_{5x} are $O(\alpha^2)$ and can be neglected to leading order; only I_{4x} need be evaluated. For the assumed spectral form, the variances describing the fluctuating relative velocities are then

$$\overline{\Delta v_y \Delta v_y} = I_{4y} = \overline{u_y u_y}\left\{\frac{\alpha\pi}{4} + O(\alpha^2)\right\} \qquad [23]$$

and

$$\overline{\Delta w_x \Delta w_x} = I_{4x} + I_{5x} + I_{3x} = \overline{u_x u_x}\left\{\frac{\alpha\pi}{4} + O(\alpha^2)\right\} \qquad [24]$$

The variances described in [23] and [24] appear in this analysis as intermediate quantities required to obtain the velocity covariances. However, the quantities are important themselves because they describe the rate at which the particle path deviates from the path of the packet of fluid that contains it. This deviation causes a loss of correlation between the motion of a particle and that of the fluid packet. As such, they provide information regarding the adequacy of using a Lagrangian spectrum to describe the fluid velocity as seen along the random path of a neutrally bouyant particle. Examination of [23] and [24] indicates two things. With or without a mean velocity gradient, the variance of the fluctuating relative velocity grows with particle Stokes number α. Consequently, linear analyses are valid for small values of the Stokes number only. In addition, the mean velocity gradient does not affect the fluctuating relative velocity variance to leading order in α.

In addition, the magnitude of the relative velocity can be important in assessing the accuracy of analyses like this one when used to predict the velocity statistics of particles that obey other dynamic equations. The particle dynamic equations described here in [1] and [2] are exact when the Reynolds number based on the particle velocity and radius are much less than 1. When the Reynolds number is small, the drag term is non-linear in the relative velocity. Lumley (1978) proposed that linearized equations similar to [1] and [2] may be used provided the two Reynolds numbers based on either the mean relative velocity or the variance in fluctuating relative velocity are small. Thus [23] and [24] also provide a measure of the degree to which dynamic equations such as [1] and [2] describe particle motions.

When the particle Stokes number $\alpha = \tau_p \omega_e$ is small, the leading order contributions for the velocity variance were shown to be (Liljegren 1990, 1993)

$$\overline{v_y v_y} = \overline{u_y u_y}\left\{1 - \alpha\frac{\pi}{4} + O(\alpha^2)\right\} \qquad [25]$$

and

$$\overline{w_x w_x} = \overline{u_x u_x}\left\{1 + \alpha\left(2\frac{|G\overline{u_x u_y}|}{\omega_e \overline{u_x u_x}} - \frac{\pi}{4}\right) + O(\alpha^2)\right\} \qquad [26]$$

To leading order, the fluctuating velocity is not affected by the existence of the mean velocity gradient. Using [8], [9], [23], [24], and the previously obtained [25] and [26], the two components of the fluid velocity covariance become

$$\overline{u_x w_x} = \overline{u_x u_x}\left\{1 + \alpha\left(\frac{|G\overline{u_x u_y}|}{\omega_e \overline{u_x u_x}} - \frac{\pi}{4}\right) + O(\alpha^2)\right\} \qquad [27]$$

and

$$\overline{u_y v_y} = \overline{u_y u_y}\left\{1 - \alpha\frac{\pi}{4} + O(\alpha^2)\right\} \qquad [28]$$

Equations [23], [24], [27] and [28] are the principal new results contained in this paper. Along with [25] and [26] they describe all relative velocity variances, fluid–particle velocity covariances and particle velocity variances. Some important features of the relative velocity variance and the fluid–particle velocity covariances are discussed in section 3.

As noted, the fluid turbulence spectrum in a flow with mean velocity gradients will be anisotropic. Moreover, elevation of the particle–fluid velocity covariance will tend to accentuate the anisotropy of the turbulence. This has two major consequences for the problem discussed here. First, the ratio $\overline{u_y u_y}/\overline{u_x u_x} \neq 1$. Using the actual velocity variances in [23]–[28] rather than $1/3\,\overline{u_i u_i}$ accounts for any possible differential distribution of energy. Second, the functional form of the velocity spectrum will differ in the two directions. In the flow described here, production of turbulent kinetic energy is expected to contribute to the streamwise, or x, component for the turbulent kinetic energy and to contribute preferentially at the large scales. Energy would be more evenly distributed at small scales. Any differences in the functional form for the velocity spectrum affect the coefficient of $\pi/4$ appearing in the four equations. Because particles are better able to respond to large scale fluctuations, the particular difference in the spectrum described here would cause the coefficient in the equations describing streamwise components to be smaller than those appearing in the equations describing transverse, or y, components.

This means that the equations can be written more generally as

$$\overline{\Delta v_y \, \Delta v_y} = \overline{u_y u_y}\{C_1 \alpha + O(\alpha^2)\} \tag{29}$$

$$\overline{v_y v_y} = \overline{u_y u_y}\{1 - C_1 \alpha + O(\alpha^2)\} \tag{30}$$

$$\overline{u_y v_y} = \overline{u_y u_y}\{1 - C_1 \alpha + O(\alpha^2)\} \tag{31}$$

$$\overline{\Delta w_x \, \Delta w_x} = \overline{u_x u_x}\{C_2 \alpha + O(\alpha^2)\} \tag{32}$$

$$\overline{w_x w_x} = \overline{u_x u_x}\left\{1 + \alpha\left(2\frac{|G\overline{u_x u_y}|}{\omega_e \overline{u_x u_x}} - C_2\right) + O(\alpha^2)\right\} \tag{33}$$

and

$$\overline{u_x w_x} = \overline{u_x u_x}\left\{1 + \alpha\left(\frac{|G\overline{u_x u_y}|}{\omega_e \overline{u_x u_x}} - C_2\right) + O(\alpha^2)\right\} \tag{34}$$

Both C_1 and C_2 will be $O(1)$ in magnitude and $C_1 > C_2$.

3. ASSESSMENT OF THE POSSIBILITY OF TURBULENCE GENERATION

Methods to predict the effect of particles on the turbulence are not well devleoped. However, it has been proposed that some qualitative features may be determined by deriving approximate equations for the transport of fluctuating kinetic energy using the following method. First, assume that a set of transport equations for the fluid and particle momentum describes the motion of interpenetrating continua. Then, decompose the velocity, volume fraction and pressure into mean and fluctuating components. Perform the scalar product between the momentum equation and a corresponding velocity fluctuation, and finally taking the ensemble average.

The manipulation described is not rigorous, primarily because a particulate mixture is not an interpenetrating medium. At best, the effect of turbulence structures that are smaller than the volume required to average out phase discontinuities cannot be captured in this way (Besnard & Harlow 1988). Consequently, it is likely that some terms describing transport of turbulent kinetic energy will not be captured. For this reason, the author does not believe that great confidence may be placed on models derived in this way. Nevertheless, it seems likely that some of the terms describing turbulence modulation by particles can be captured in this way, and the procedure appears to capture the physical mechanism describing transfer of turbulent kinetic energy between phases and dissipation of trubulent kinetic energy as a result of particle–fluid interactions. This procedure will be applied below; for simplicity, the volume fraction will be assumed constant.

Numerous transport equations for the particle and fluid phases of a particulate mixture have been proposed. Differences appear in the form of the pressure terms, added mass coefficients and other terms. Despite these differences, many of the forms are equally useful for the purpose of illustrating the effect of the covariance and fluctuating relative velocity on turbulence modulation by particles, which is most strongly affected by the appearance of an interfacial drag term in the averaged equations. If the following general transport equations for fluid and particle momentum are accepted as correct:

$$\frac{\partial \rho_p \Phi_p V_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho_p \Phi_p V_i V_j) = -\rho_p \Phi_p \beta(V_i - U_i) - \phi_p \frac{\partial P^p}{\partial x_i} + \rho_p \Phi_p g_i \tag{35}$$

$$\frac{\partial \rho_f \Phi_f U_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho_f \Phi_f U_i U_j) = \rho_p \Phi_p \beta(V_i - U_i) + \Phi_f\left\{-\frac{\partial P^f}{\partial x_i} + \mu_f \frac{\partial}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right\} + \rho_f \Phi_f g_i \tag{36}$$

where ϕ_f and ϕ_p are the fluid and particle volume fractions and P^p and P^f are pressure in the fluid and particle phases, respectively.

Then the transport equation for fluctuating kinetic energy of both phases obtained using the procedure outlined above would be

$$\frac{1}{2}\frac{D\rho_p\phi_p\overline{v_iv_i}}{Dt}+\frac{\partial}{\partial x_j}(\rho_p\phi_p\overline{v_iv_j}V_i)=\underbrace{-\rho_p\phi_p\beta\overline{v_i(v_i-u_i)}}_{\text{dissipation and transfer}}-\underbrace{\phi_p\frac{\partial}{\partial x_i}\overline{p^pu_i}}_{\text{pressure gradient work}} \quad [37]$$

production

$$\frac{1}{2}\frac{D\rho_f\phi_f\overline{u_iu_i}}{Dt}+\frac{\partial}{\partial x_j}(\rho_f\phi_f\overline{u_iu_j}U_i)=\underbrace{\rho_p\phi_p\beta\overline{u_i(v_i-u_i)}}_{\text{dissipation and transfer}}-\underbrace{2\nu\overline{s_{ij}s_{ij}}}_{\text{dissipation}}$$

production

$$-\left\{\underbrace{\phi_f\frac{\partial}{\partial x_i}\overline{p^fu_i}}_{\text{pressure gradient work}}+\underbrace{\frac{1}{2}\phi_f\overline{u_iu_iu_j}}_{\text{diffusion}}-\underbrace{2u\phi_f\overline{u_is_{ij}}}_{\text{transport by viscous stresses}}\right\} \quad [38]$$

where $s_{ij}=\partial u_i/\partial x_j$ and v represents the particle velocity in an Eularian framework.

According to this formulation, the total effect of the particles on the transport of fluctuating kinetic energy for the mixture can be obtained by adding the two terms labeled "dissipation and transfer"; the sum is defined as $-\epsilon_{pT}=-\rho_p\phi_p\beta\,\overline{\Delta v_i\,\Delta v_i}$. This term has an obvious physical interpretation. It describes irreversibilities associated with energy transfer between phases; as such it is always negative. In terms of the variables used in the analysis of particle moments contained in section 2 this total dissipation is

$$-\epsilon_{pT}=-\rho_p\phi_p\beta(\overline{\Delta w_x\,\Delta w_x}+\overline{\Delta v_y\,\Delta v_y}+\overline{\Delta v_z\,\Delta v_z}) \quad [39]$$

It is possible to extend the analysis described in section 2 to include the z component, using a method similar to that shown in section 2. Substituting [23], and [24] the overall effect of particles on the turbulence transport then becomes

$$-\epsilon_{pT}=-\rho_p\phi_p\beta\frac{\alpha\pi}{4}(\overline{u_xu_x}+\overline{u_yu_y}+\overline{u_zu_z}) \quad [40]$$

The mean velocity gradient, G, does not appear in [40]. This indicates that the existence of a mean velocity gradient does not affect the dissipation of turbulent kinetic energy, at least when the particles contained in the mixture are small.

However, the effect on the individual phases is more complicated. In the fluid equations, the effect of the particles appears through $\epsilon_{pf}=\rho_p\phi_p\beta\overline{u_i(v_i-u_i)}$. Analyses based on stationary homogeneous isotropic turbulence (HIT) indicate that $\overline{u_iv_i}\ll\overline{u_iu_i}$. Consequently, this term would always be negative in these flows, and particle–fluid interactions would act as a turbulence "sink" in the equation describing transport of turbulent kinetic energy for the fluid phase. However, in the presence of a velocity gradient, this term may change sign and act as a "source" for turbulence in the fluid phase.

The role of the particles can be explained as follows. In the presence of a velocity gradient, energy is exchanged between the mean and fluctuating fields of both phases through the action of the Reynolds stresses. So fluctuating kinetic energy is created in both phases. Ultimately, all fluctuating kinetic energy is dissipated through two mechanisms. The first is viscous dissipation at small scales which appear to occur only in the fluid phase, and also occurs in single-phase flow. The second mechanism is irreversible losses when work is performed at the particle fluid interfaces; the magnitude of this loss is described by $-\epsilon_{pT}$ [39].

Although the overall action of the particle-fluid interaction is dissipative, the effect of particle–fluid interactions can be to act as a source of turbulent kinetic energy in the fluid phase. According to [38], this occurs when the fluid–particle velocity covariance is greater than the fluid velocity variance. Physical arguments would indicate that it must occur whenever fluctuating kinetic energy cannot be dissipated as rapidly as it is generated in the particulate phase; when this occurs some of the fluctuating kinetic energy created in the particle phase must be transferred to the fluid phase where it may be dissipated to heat through viscous action. It appears that this can occur in the presence of a mean velocity gradient.

4. CONCLUSION

Analysis of the effect of a mean velocity gradient on the particle–fluid velocity covariance suggests that particle–fluid interactions can act as a source for turbulence in the fluid phase. This behavior is qualitatively different from that expected in their absence. The results of this analysis suggest that models that are used to predict the effect of particles on fluid turbulence should generally account for the presence of mean velocity gradients. Failure to capture the effect of mean velocity gradients will, at a minimum, overpredict the apparent dissipation of turbulent kinetic energy in the fluid phase. It will also fail to detect regions in which turbulent kinetic energy is transferred from the particle to the fluid phase as a result of particle–fluid interactions; this transfer causes the term describing the effect of interactions between the fluctuating velocities in both phases to appear as a source term of turbulent kinetic energy in the fluid phase.

It is the author's opinion that methods to capture the effect of the mean velocity gradient on the magnitude of the fluid–particle velocity covariance and the related particle velocity variance can be developed within any general framework for turbulence modeling of particulate flows, ranging from semi-Lagrangian methods like those of Migdal & Acosta (1967), or Oesterle & Petitjean (1993) to fully Eulerian methods that extend the work of Elghobashi & Abou-Arab (1983), or Kataoka & Serizawa (1989). However, the author cautions against using [27] and [29] derived here as algebraic closures for the covariance in any proposed turbulence models. This is because features like boundaries, second derivatives in the mean velocity and other complications that were ignored above, are likely to affect the magnitude of the covariance in many real flows.

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